Louvain Institute of Data Analysis and Modeling in economics and statistics (LIDAM)
Center for Operations Research and Econometrics (CORE)

Forward electricity markets with nonconvexities

Quentin Lété[1], Thomas Hübner[2]

[1] UCLouvain [2] ETH Zürich

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Section 1. Pricing on electricity markets

Convex case - example

Consider

- 3 generators

	Generator 1	Generator 2	Generator 3
Min generation [MW]	0	0	0
Max generation [MW]	40	25	15
Marginal cost [€/MWh]	20	36	50
Start-up cost [€]	0	0	0

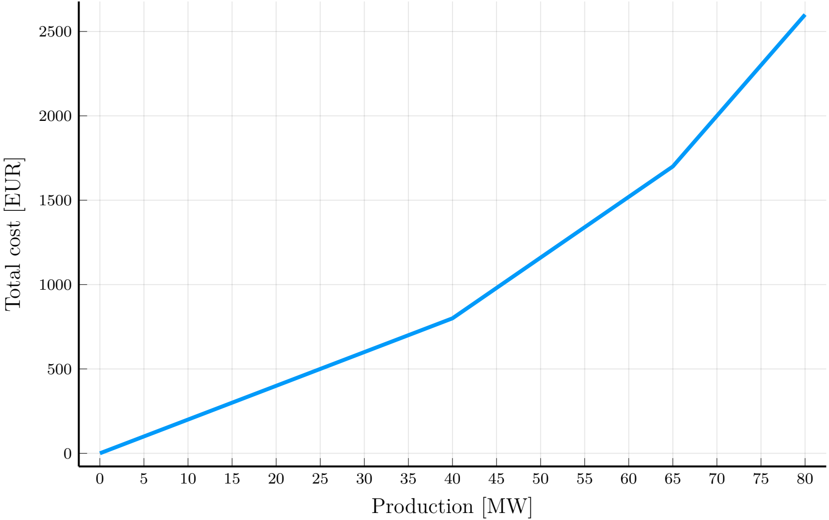
- Inflexible demand at 45 MW

Question: What is the competitive equilibrium?

That is: **quantities** and **price** s.t.

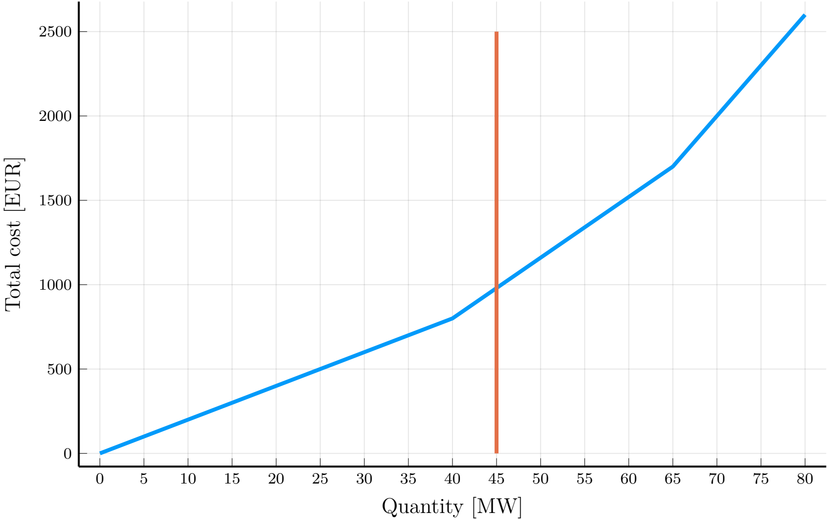
- Each agent maximizes their utility given the price
- Supply equals demand

Total (minimum) cost curve:



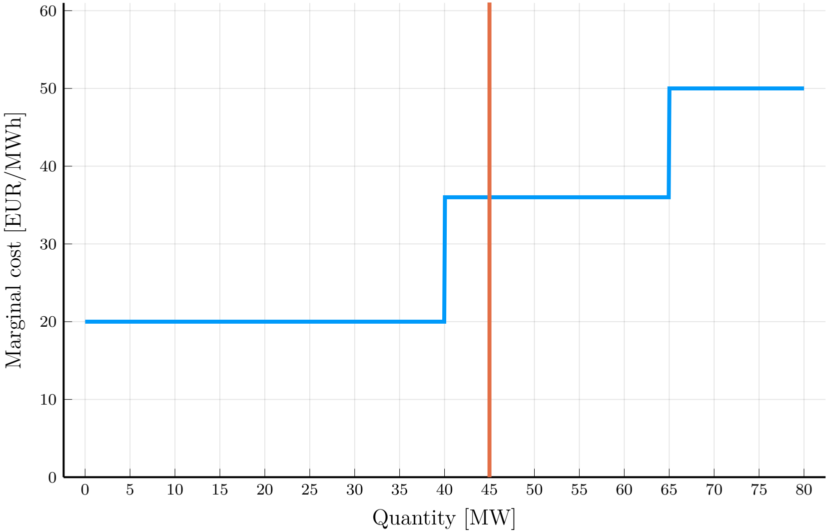
Convex case - example

Total cost curve:



Convex case - example

Marginal cost curve:



In the day-ahead market, costs are nonconvex

Reasons:

- Start-up costs
- Minimum up-time and down-time
- Ramping constraints

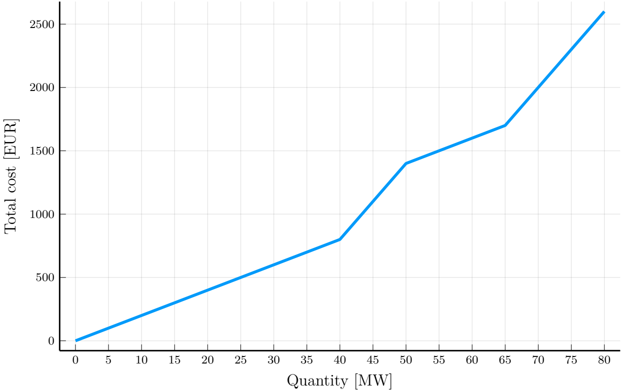
Example:

	Generator 1	Generator 2	Generator 3
Min generation [MW]	0	25	0
Max generation [MW]	40	25	15
Marginal cost [€/MWh]	20	0	50
Start-up cost [€]	0	900	0

Nonconvex case - example

	Generator 1	Generator 2	Generator 3
Min generation [MW]	0	25	0
Max generation [MW]	40	25	15
Marginal cost [€/MWh]	20	0	50
Start-up cost [€]	0	900	0

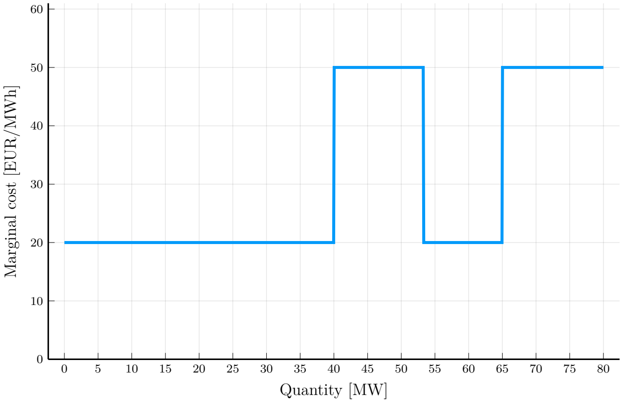
→ Total cost curve becomes **nonconvex**



Nonconvex case - example

	Generator 1	Generator 2	Generator 3
Min generation [MW]	0	25	0
Max generation [MW]	40	25	15
Marginal cost [€/MWh]	20	0	50
Start-up cost [€]	0	900	0

→ Marginal cost curve becomes **non-monotonic**



Absence of competitive equilibrium

Consequence of nonconvexity

- Marginal price does not lead to a competitive equilibrium
- Some agents might be making losses at marginal price

E.g., generator 2:

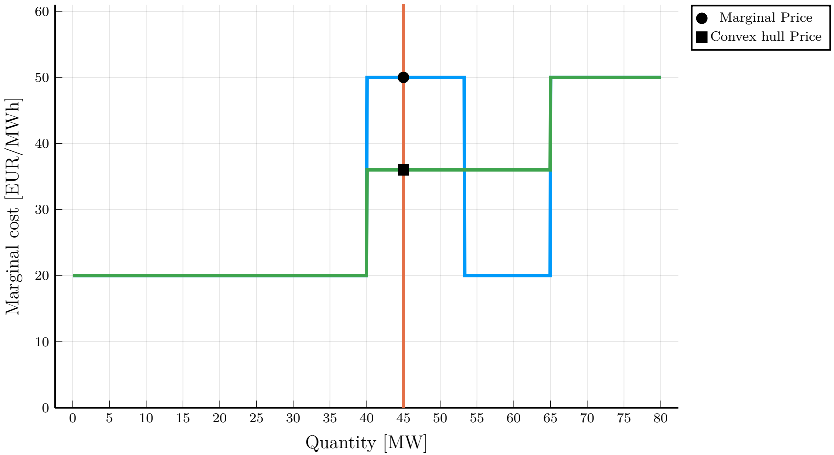
- If it follows optimal solution: Profit = 0€
- If it does not follow optimal solution: Profit = $25 \cdot 50 - 900 = 350$€

$\rightarrow$ Some agents are better-off by deviating from welfare maximizing solution

Solution

- Side payments
- Agents receive the difference between their max profit given the price and their profit given the price and the optimal solution

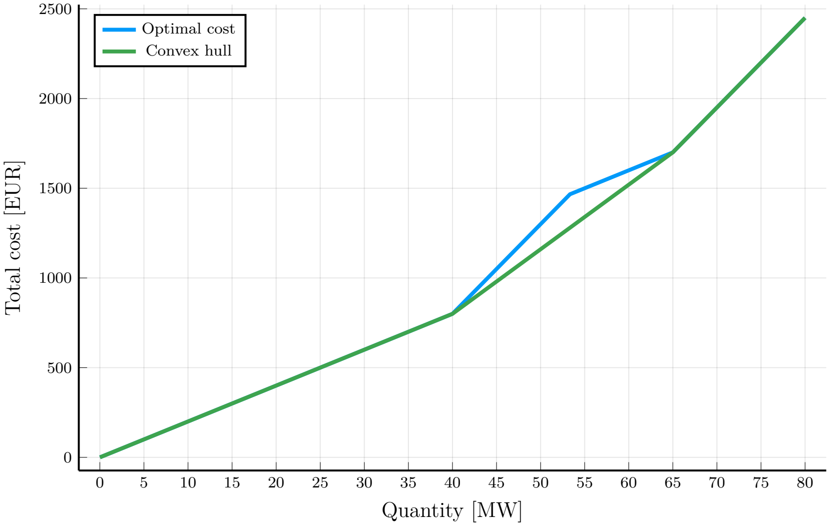
Two possible pricing methodologies



Theorem

Convex hull pricing minimize side payments (Gribik et al., 2007)

Convex hull


Optimal cost
Convex hull
Total cost [EUR]
Quantity [MW]
0
500
1000
1500
2000
2500
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80

The day-ahead is a forward market

→ Effect of Virtual trading

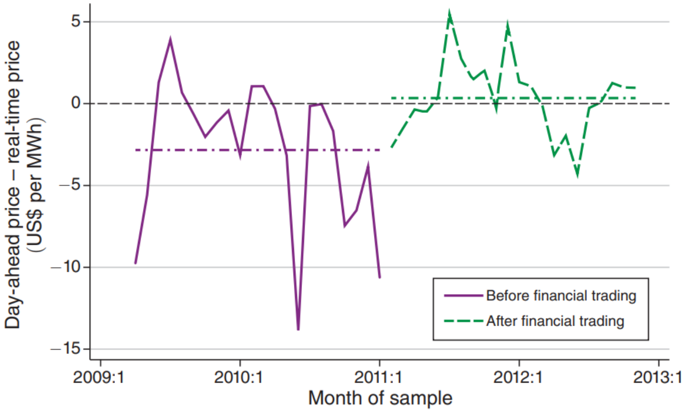


Figure: Monthly average Day-Ahead price minus real-time price in California (April 2009 - November 2012). Source: Jha and Wolak (2023)

Combining forward market and nonconvexity: open questions

Observations

- If no uncertainty and marginal pricing is used in day-ahead, financial participants make no profit.
- If convex hull pricing is used, they could make a profit.
- Even though they do not bring any benefit to the system (when no uncertainty).

Questions

- Can the action of financial participants deteriorate welfare?
- With financial participants, is it still true that convex hull pricing minimizes the side payments?
- Empirically, can this have a real effect on the market?

Section 2. Model

Model

Setting

- Assume financial participants have perfect knowledge of the market
- Convex hull pricing in day-ahead
- Denote y the quantity traded by financial participants in the day-ahead market
- Define three optimization problems based on y

$$
\begin{aligned}
WELFARE(y) = \\
\min_{p_g,u_g} \ & \sum_g MC_g p_g + SC_g u_g \\
\text{s.t.} \ & \sum_g p_g + y = D \\
& \underline{P}_g u_g \le p_g \le \overline{P}_g u_g \quad \forall g \\
& u_g \in \{0,1\} \quad \forall g
\end{aligned}
$$

$$
\begin{aligned}
DAY-AHEAD(y) = \\
\min_{p_g,u_g} \ & \sum_g MC_g p_g + SC_g u_g \\
\text{s.t.} \ & \sum_g p_g + y = D \quad [\lambda^{DA}] \\
& \underline{P}_g u_g \le p_g \le \overline{P}_g u_g \quad \forall g \\
& p_g, u_g \in \text{conv} \begin{pmatrix} \underline{P}_g u_g \le p_g \le \overline{P}_g u_g \quad \forall g \\ u_g \in \{0,1\} \quad \forall g \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
REAL-TIME(u_g^*) = \\
\min_{p_g,u_g} \ & \sum_g MC_g p_g + SC_g u_g \\
\text{s.t.} \ & \sum_g p_g = D \quad [\lambda^{RT}] \\
& u_g = u_g^* \quad \forall g
\end{aligned}
$$

Model

- Maximize profit of financial participation

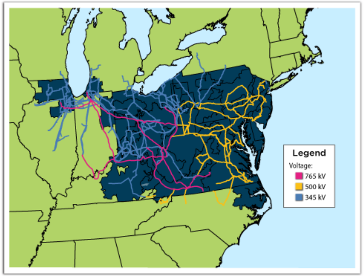
$$\begin{aligned} \max_{y} \quad & y(\lambda^{DA} - \lambda^{RT}) \\ \text{s.t. } & u_g^* \in WELFARE(y) \\ & \lambda^{DA} \in DAY-AHEAD(y) \\ & \lambda^{RT} \in REAL-TIME(u_g^*) \end{aligned}$$

With $\in$ meaning "is an optimal (primal/dual) solution"

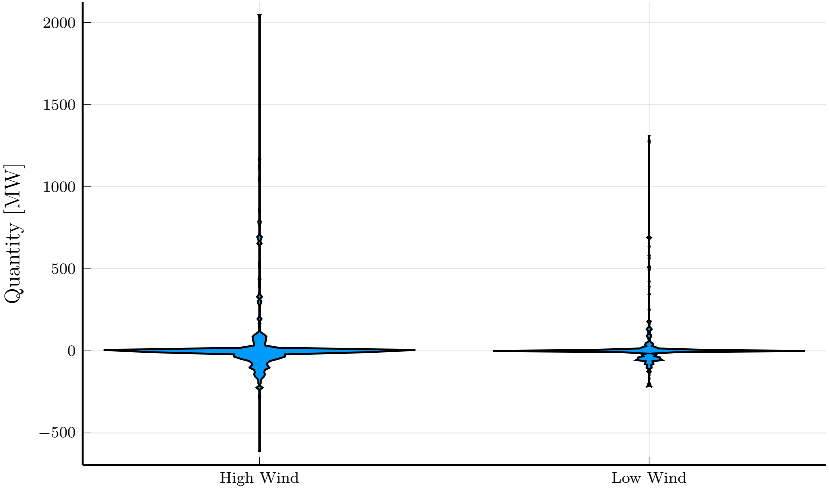
Section 3. Case study: the PJM market

Case study overview

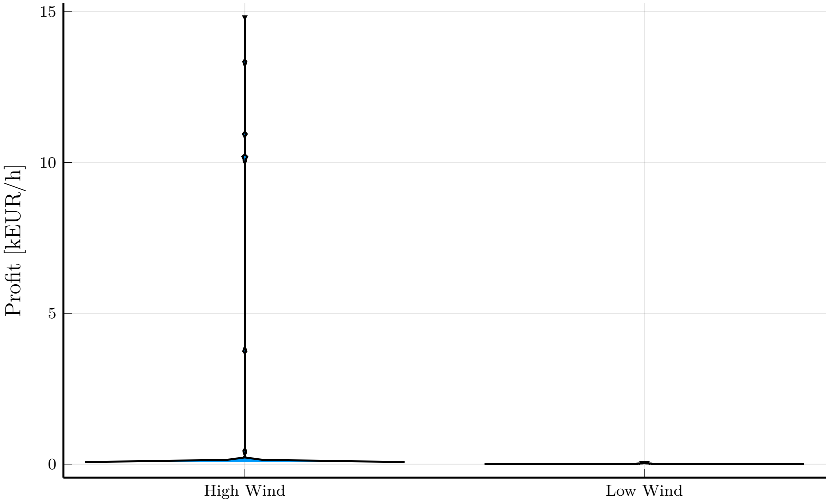
- Publicly available generator data from FERC.
- Publicly available load, reserves, and wind data from PJM.
- ’lw’: a wind profile scaled to be 2% of annual load;
- ’hw’: a wind profile scaled to be 30% of annual load.
- 10 days with 48 time periods each from early january.



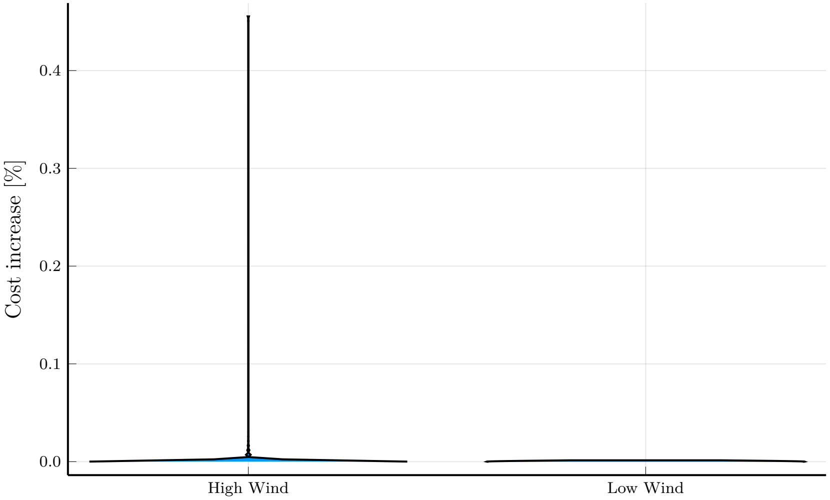
Case Study results: optimal quantity traded



Case Study results: optimal profit



Case Study results: cost increase



Section 4. Discussion and conclusion

Discussion and conclusion

Discussion

- Situation where financial participants perfectly coordinate and have perfect information on the market.
- Virtual bidding not allowed in the EU, but still ways to do it.
- Locational and temporal pricing would increase the effects described.

Conclusion

- When viewed in a forward market context, the theoretical properties of convex hull pricing disappears:
  - Welfare maximizing
  - LOCs minimizing
- On realistic data, quantity traded and profit is substantial, but the cost increase remains moderate.
- Expected to increase with renewable penetration.

Thank You!

Contact :
Quentin Lété, quentin.lete@uclouvain.be
https://qlete.github.io